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Solving of a projection problem for convex polyhedra given by a system of linear constraints

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Abstract

© 2017 IEEE. We propose a novel approach to solving the problem which is referred to as the polyhedral projection problem (PPP) and serves to find a projection of a point onto a polyhedron given by the linear inequality constraints. The basic idea of this approach is to utilize a reduction of the PPP to the problem of projecting the origin of Euclidean space onto the Minkowski difference of the considered polyhedron and point. We make use our previous results related to the concept of the Minkowski difference for the above-mentioned objects. The proposed approach is new (relative to the traditional ones) thanks to further reducing the PPP to the problem of projecting the origin onto the convex hull of some vectors corresponding to the gradients of the constraint functions. In the paper, this reduction is justified for the case when all of constraints of the PPP are violated at the point being projected onto the originally given polyhedron. In this case, the presented reduction makes broader a spectrum of the powerful tools of mathematical programming which may be operated for solving the PPP.

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